Galileo’s projectile model

In this activity you will validate Galileo’s model for the motion of a projectile, by comparing the results predicted by the model with results from your own experiment.

Information sheet: Projectiles

Often motion does not take place in a straight line. One of the most common kinds of motion in two dimensions is that of an object which has been thrown or projected, and then moves freely under the influence of gravity. These objects are referred to as *projectiles*.

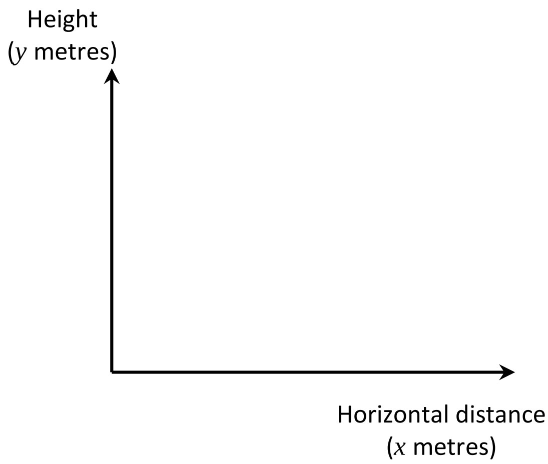
For example, if you kick or hit a ball it can be modelled as a projectile by ignoring any forces acting on the ball other than the weight.

Think about:

What assumptions are being made by modelling an object as a projectile?

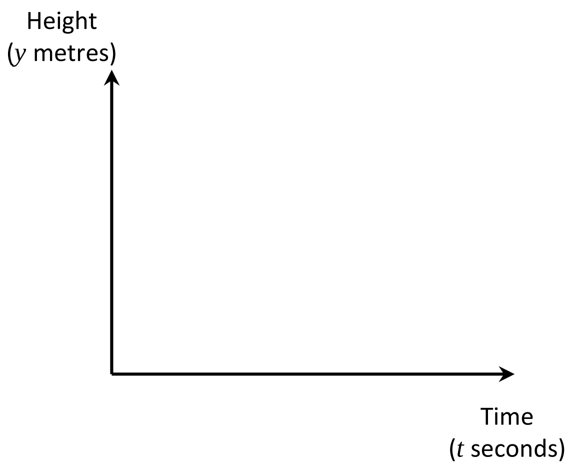
Observing a projectile

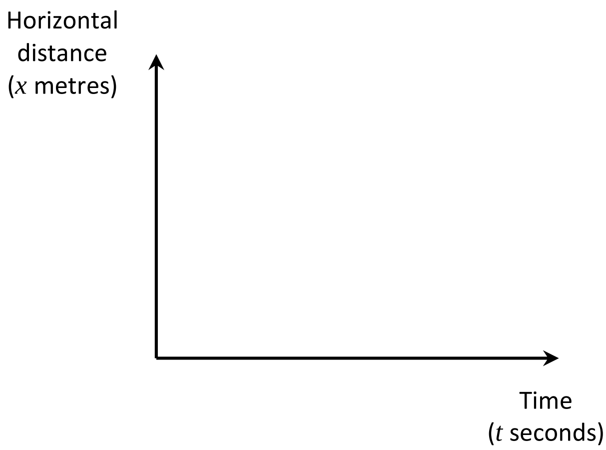
Use the axes *on the right* to sketch a graph of the path of a ball thrown gently across a room.



Now use your observations to sketch graphs of the following, on the two axes given *below*:

- the height, y metres, against the time, t seconds
- the horizontal distance travelled, x metres, against the time, t seconds.





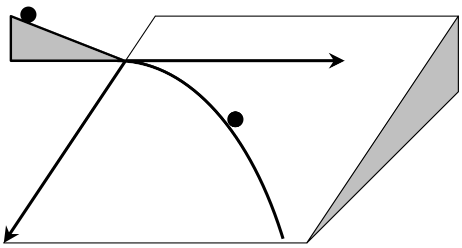
Think about

Which feature of a distance–time graph represents speed?

Information sheet: Galileo's projectile model

It is difficult to collect data which will allow you to plot the graphs with a reasonable degree of accuracy, especially using only simple apparatus.

When Galileo investigated the motion of a projectile, he deflected the motion of a ball down a slope as shown here.

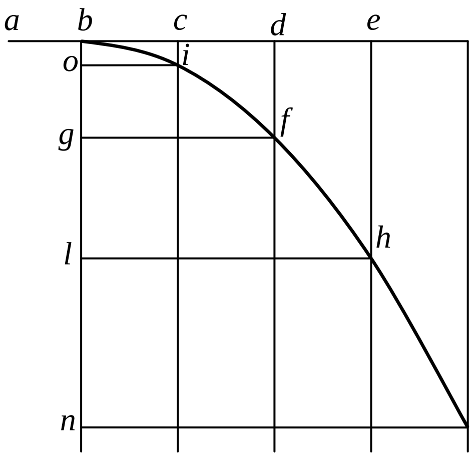


In a range of experiments, Galileo investigated the relationships between distance covered and time taken, but he investigated horizontal and vertical distances separately.

Through these experiments, Galileo established that the motion of a projectile is a combination of constant horizontal velocity and vertical motion, in which the projectile accelerates at a rate of 9.8 m s^{-2}.

This means that the horizontal distance travelled is proportional to t, the time taken; and provided the projectile is launched horizontally, the vertical distance travelled is proportional to t^2.

Galileo did not have access to the algebra used today. He had to resort to writing about a diagram such as this.



The curved line represents the path of the projectile launched at b.

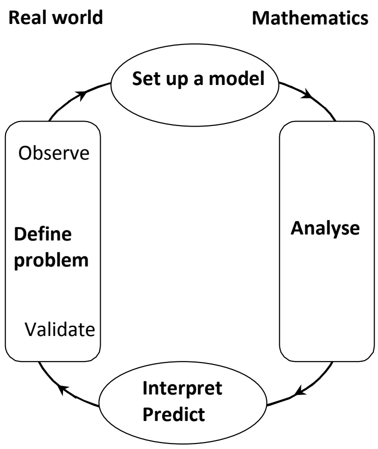
The points i, f, h give the position of the projectile after equal time intervals.

Think about

What can you say about the distances bc, cd, and de?
What does this tell you about the horizontal velocity of the ball and the horizontal distance covered by the ball?

How could you check that the vertical distances are proportional to t^2?

Analysing and validating Galileo's model



The modelling cycle is shown in this diagram. In this activity the model, Galileo's projectile model, has already been set up for you and you will use it to analyse the motion of a projectile and predict how far a projectile will travel. You will also validate the model by comparing your predictions with your experimental results.

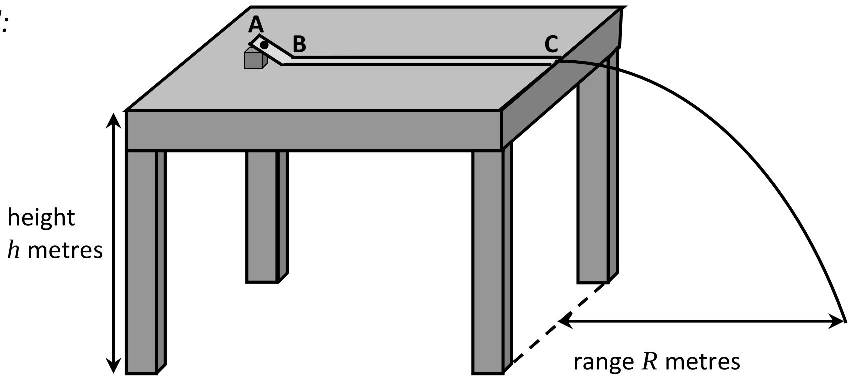
Analysing

Galileo's model allows the equations for motion in a straight line with constant acceleration, $s = \frac{(u+v)t}{2}$, $v = u + at$, and $s = ut + \frac{1}{2}at^2$, to be applied separately to the horizontal and vertical motion of projectiles.

You will use these equations to predict the horizontal distance travelled by a projectile launched horizontally with speed u.

Validating

For the practical activity you will need:
track, block, ball, ruler
stopwatch
talcum powder or salt (optional)



You will use the apparatus as shown in the diagram to project a ball into the air horizontally.

Think about:

What modelling assumptions will you make about

- the ball?
- the speed along BC?
- the forces acting on the ball?
- the path of the ball?

What are the constants?

What are the variables?

What will affect the horizontal distance R covered by the ball?

How can you control the speed of projection at C?

To estimate the launch velocity, assume that the velocity of the ball along BC is constant, and that this is the velocity of the ball as it is launched horizontally at C.

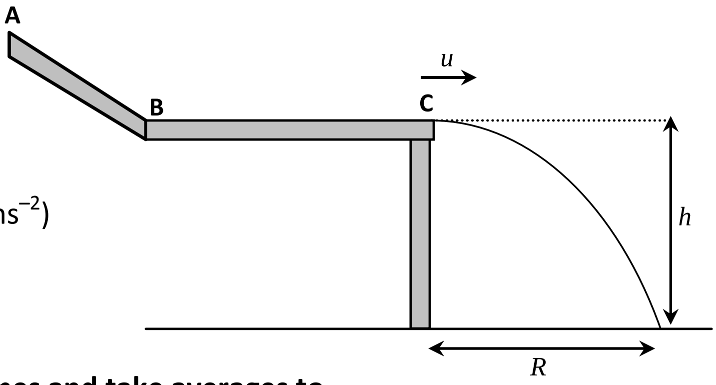
Launch velocity = average velocity along BC = $\dfrac{\text{distance travelled}}{\text{time taken}}$

Think about

How does this use the assumption that velocity is constant along BC?

Try these

1 Copy this diagram to show the motion of the projectile in a plane.
Record the value of h for your table.



u = velocity of launch
g = acceleration due to gravity (take g = 9.8 ms^{-2})
h = height of table
R = range of projectile

In task 2, repeat each experiment several times and take averages to improve the accuracy of your results.

2 For different release points along the slope AB:

a record the time the ball takes to roll along BC

b use average velocity along BC = $\dfrac{\text{distance travelled}}{\text{time taken}}$ to estimate the launch velocity

c measure and record the horizontal range, R, of the ball.
(To find where the ball lands, you may find it useful to use talcum powder or salt sprinkled on a large sheet of paper or in a baking tray.)

d Use your experimental results to draw a graph of R against u.

In tasks 3 and 4, use the uniform acceleration equation $s = ut + \frac{1}{2}at^2$ for the vertical and horizontal motion of the projectile.

3a Write down an equation for the vertical distance travelled, y, in terms of g and t.

b Use your equation and the height of the table to predict the time taken for the ball to land.

4a Write down an equation for the horizontal distance travelled, x, in terms of u and t.

b Use your equations to find a relationship between R, u, and h.

c Sketch a graph of R against u for this relationship.

5a Calculate values of R for the value of h in your experiment and a range of appropriate projection speeds, u.

b Draw an accurate graph of R against u.

6 Use your graphs from **5b** and **2d** to compare the predicted range of a projectile with your experimental results:

a How well does your experimental data fit the predictions?

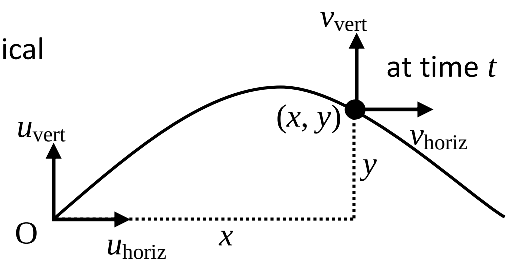
b Do the predictions give an underestimate or an overestimate for the range of the projectile?

c Explain why there may be discrepancies between the two graphs.

Extension

So far in this activity you have only considered horizontal projection. Galileo's model also applies to projection which is not horizontal, that is projectiles launched with a non-zero vertical component of velocity.

In this diagram, u_{vert} and u_{horiz} are the horizontal and vertical velocities of projection. v_{vert} and v_{horiz} are the horizontal and vertical velocities of the projectile after time t. x is the horizontal distance travelled and y is the height above the point of projection.



1 Using Galileo's model, write down equations for v_{horiz}, and x in terms of u_{horiz}, and the time, t.
Also write down equations for v_{vert} and y in terms of u_{vert} and the time t.
(Think about the direction in which the projectile is accelerating.)

2 Use your equations to sketch graphs to show how v_{horiz}, v_{vert}, x and y depend on t. How do your graphs for x and y compare with those you sketched in the introduction to this activity?

Reflect on your work

What are the advantages of Galileo's projectile model?

Do your experimental results validate Galileo's projectile model?

Can you suggest examples of motion that could not be modelled very well as projectiles?